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Introduction and Basic Concepts

The kinematics and dynamics of multibody systems is an important part of what is referred to as CAD (*Computer Aided Design*) and MCAE (*Mechanical Computer Aided Engineering*). Figures 1.1 to 1.6 illustrate some practical examples of computer generated models for the simulation of real multibody systems. The mechanical systems included under the definition of *multibodies* comprise robots, heavy machinery, spacecraft, automobile suspensions and steering systems, graphic arts and textile machinery, packaging machinery, machine tools, and others. Normally, the mechanisms used in all these applications are subjected to large displacements, hence, their geometric configuration undergoes large variations under normal service conditions. Moreover, in recent years operating speeds have been increased, and consequently, there has been an increase in accelerations and inertia forces. These large forces inevitably lead to the appearance of dynamic problems that one must be able to predict and control.

The advantage of computer simulations performed by CAD and MCAE tools is that they allow one to predict the kinematic and dynamic behavior of all types of multibody systems in great detail during all the design stages from the first design concepts to the final prototypes. At any design stage, computer-aided analysis is an auxiliary tool of great value, providing a sufficient amount of data for the engineer to study the influence of the different design parameters, since it allows him to carry out a large number of simulations quickly and economically.

The *analysis* programs simulate the behavior of a multibody system once all of its geometric and dynamic characteristics have been defined. The analysis programs are certainly very useful. At the present time they are the only general purpose tools available for the largest number of applications. We are also witnessing the advent of the *design* programs that will not only perform system analyses, but also modify automatically its parameters so as to obtain an optimal behavior. An intermediate step between the analysis and optimal design programs are the *parametric* analyses, which determine the different responses of a multibody system with respect to the variation of one of the design variables. In any case, the analysis programs constitute the basis of the design programs. This book is particularly oriented towards the study of the analytical methods and numerical algorithms that are necessary to build such simulation tools. Nevertheless, we will also pay attention to some important design issues.

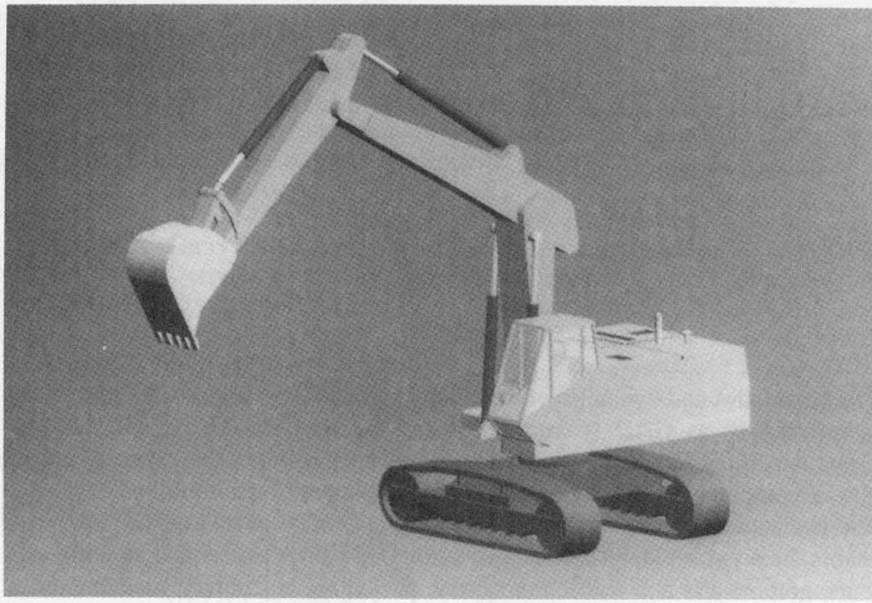

Figure 1.1. Computer model of heavy machinery.

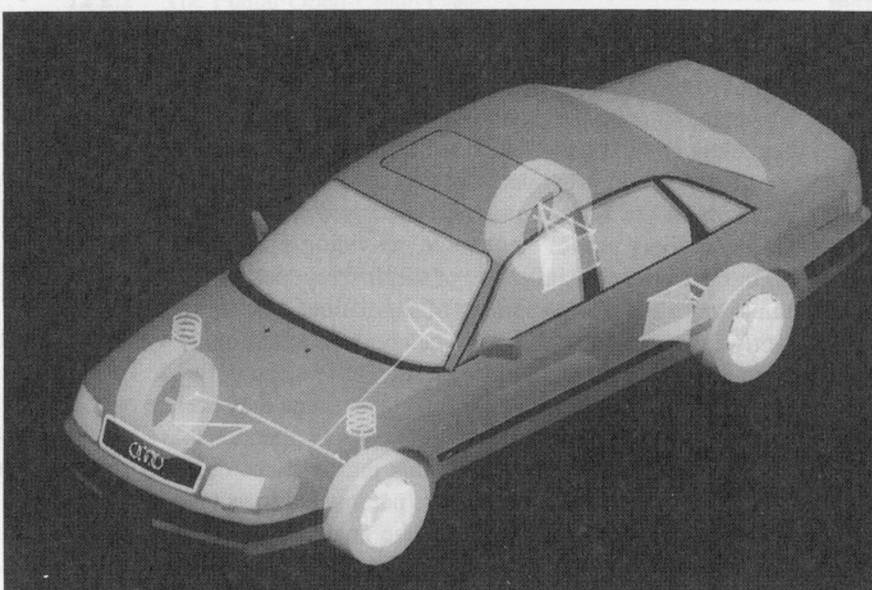

Figure 1.2. Computer model of the steering and suspension system of a car.

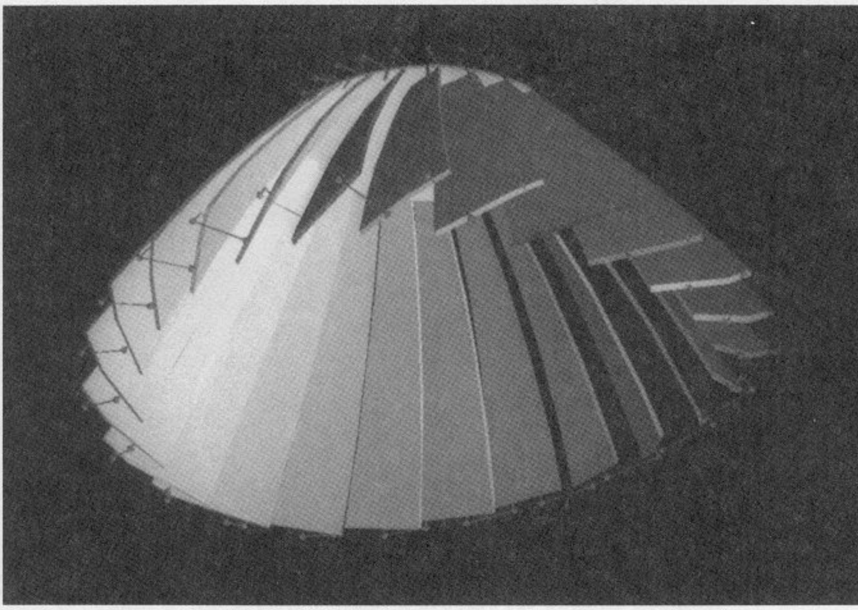

Figure 1.3. Computer model of a space deployable antenna.

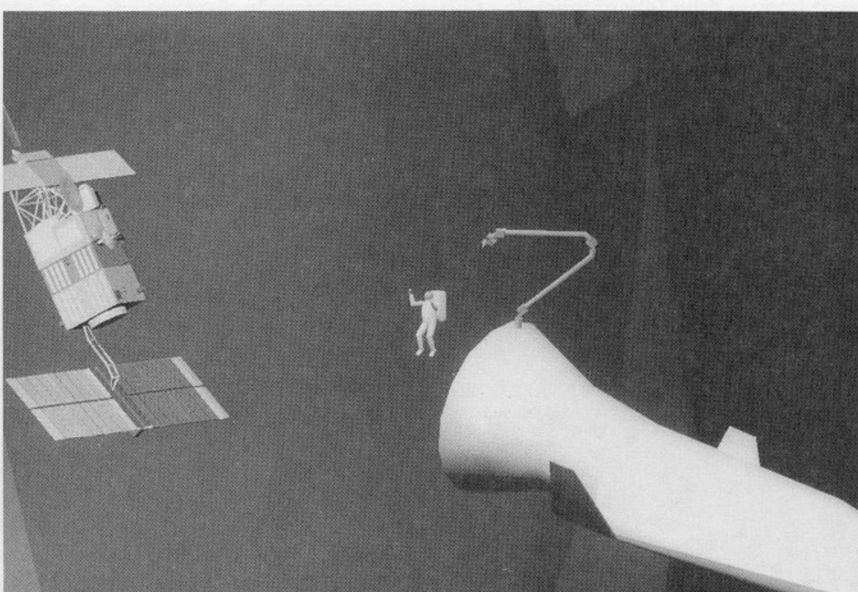

Figure 1.4. Computer model of a complex space scenario, including a satellite, the earth, a shuttle with a robot, and an astronaut.

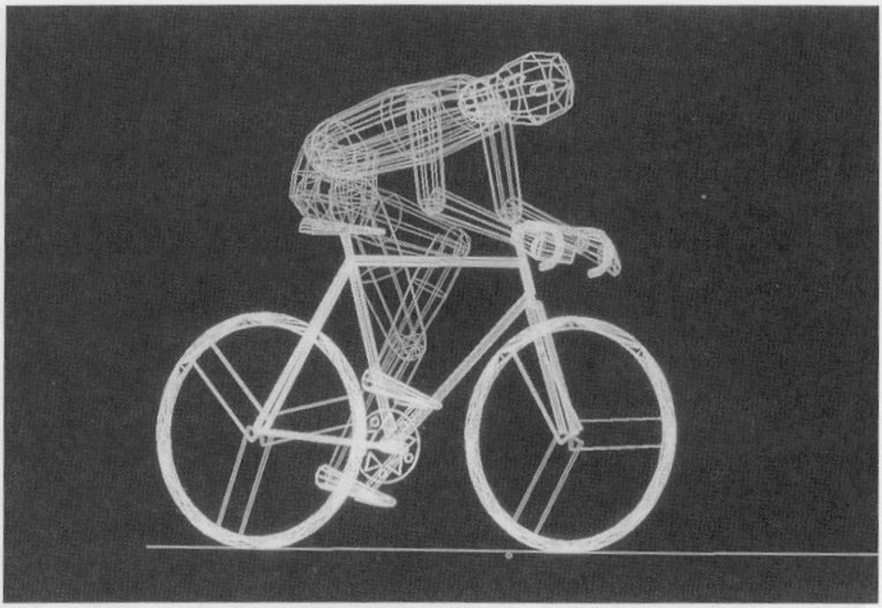

Figure 1.5. Computer model of a bicycle and a cyclist.

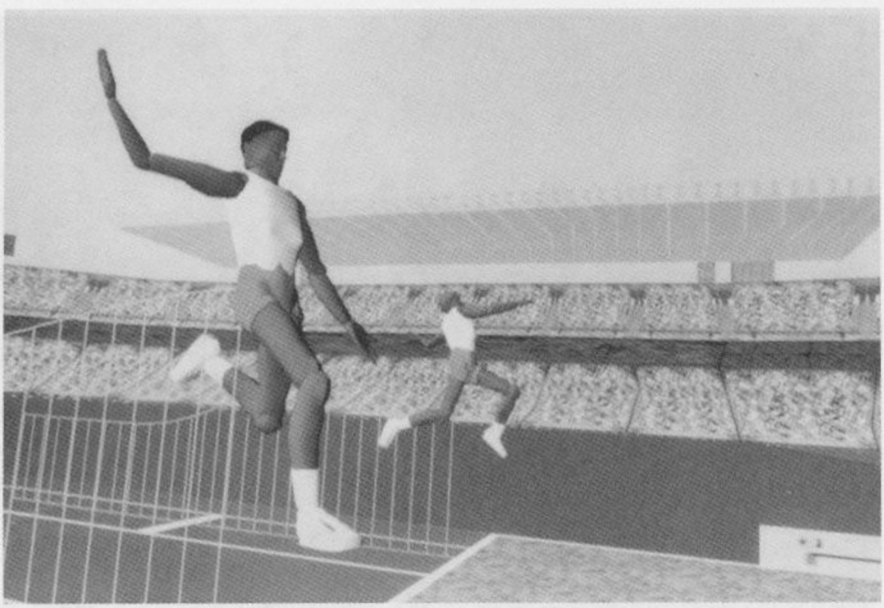

Figure 1.6. Computer animation of two long jump finalists in the 1992 Barcelona Olympic Games.

1.1 Computer Methods for Multibody Systems

Computer systems, while increasing tremendously in power in recent years, are so affordable nowadays, that their use has become widely spread in many different fields and for an immense amount of applications. Today we consider the computer as a necessary tool, whose availability is taken for granted by engineers, scientists, businessmen, writers, and others. We can take the PC as an example of a system currently used by students in the classroom, laboratory and at home, whose power exceeds that of the mainframes used in the sixties and seventies, and which only the largest corporations could afford. Engineers working for consulting firms or large corporations in the analysis or design of new products, perform their work using personal workstations. These workstations have a capability for number crunching that vastly exceeds that of minicomputers, which only a few years ago used to be considered powerful enough to satisfy the needs of a whole engineering department. The field of mechanical engineering has not been an exception to this trend. There is an increasing demand for faster executions and better graphical interfaces that will facilitate and improve the tedious tasks of data entry and interpretation of the results. The help of the computer is sought in the decision-making process for optimal designs.

The two authors of this book started their professional careers in the late seventies working on the finite element method in Spain and the United States. At that time, the analysis of a medium size finite element model with several thousands of degrees of freedom, or the complete dynamic analysis of a multibody system, could last for over twelve hours in a mainframe computer. The analyst would spend a long time preparing the data entry consisting of large numbers of punched cards and interpreting the results shown on the endless pages of computer output. This process has changed quite a bit up to the present method. The analyst can prepare the input data in an interactive manner with the help of a preprocessor running in sophisticated graphic terminals. The execution time has been reduced to a fraction of an hour of CPU of modern workstations. For larger problems the analyst considers the access to super computers or parallel architectures remotely connected to his personal system. The tedious work of interpreting the pages and pages of computer output has been alleviated and even made pleasant through the use of graphic terminals which can show an animated picture of the results.

Although *finite element analysis* and *multibody simulation* are part of the MCAE family, they are substantially different not only in their respective aims but in their *modus operandi*, namely, in the way they work. Finite element analysis must be fast. It is essentially a *batch* process, in which the user does not usually interact with the computer analysis from the beginning to the end of that process. On the other hand, the kinematic and dynamic analyses of multibody systems are processes which are most appropriately performed using *interactive analysis*. The analyst is interested in visualizing a whole set of successive responses of the multibody system, with a simulation of its behavior and operation over all the workspace and over a certain period of time. In certain cases it may be necessary to obtain a *real-time* response, and introduce the analyst as an additional element in the simulation, called *man-in-the-loop*, who may act by introducing external forces or control over specific degrees of freedom. This obviously imposes constraints on the computer hardware and software, which exceed those imposed by the finite element analysis. Real-time analysis now requires the use of mostly top of the line workstations, and is not yet possible for the very large problems. Interactive and real-time analysis will help the engineer optimize productivity and the use of his own time, which is really the most expensive part of the simulation process. Obviously the class and size of problems that may be solved in real time will increase as the computer hardware and numerical algorithms improve in the ensuing years. In any case, readers will find that the methods described in this book will always help to speed up and improve the interactive analysis of multibody systems.

The advent of powerful workstations in the computer market is making this interactive analysis now possible for the engineering profession in general and for the multibody system analysis in particular. These workstations can currently reach 100 Mips and 20 Mflops of processing power, and draw hundreds of thousands of three-dimensional vectors and polygons per second. They run under standard operating systems and graphic interfaces such as UNIX, X-Windows, MOTIF, PHIGS, etc., and may be obtained at very affordable prices. Given the rate at which the computer hardware has been improving in the past, we can only expect better and faster hardware platforms in years to come. As a consequence, it is foreseeable that the use of general purpose computer programs for the interactive three dimensional analysis of multibody systems will be considered by the engineering profession not only as a necessary tool but also as something to be taken for granted in the design process. We intend to describe in this book formulations and numerical methods aimed at this end.

Traditional methods of analysis, such as *graphical* and *analytical*, may be limited when they are applied to complicated problems. Graphic methods, although they provide a good understanding of the kinematics, lack accuracy and tend to be time-consuming. These are the reasons why they are not used for repetitive or three-dimensional analyses. Analytical or closed-form methods can be extremely efficient, although they are application-dependent, and may suffer from an excessive complexity in a multitude of practical problems.

An alternative to overcome these limitations is to resort to *numerical analysis* and the fast processing of alphanumeric data available in current digital computers. Several books have recently appeared (Nikravesh (1988), Roberson and Schwertassek (1988), Haug (1989), Shabana (1989), Huston (1990), and Amirouche (1992)) that emphasize the use of formulations and computational methods for multibody dynamic simulation. Various general purpose programs for multibody kinematics and dynamics (Schiehlen, (1990)) have been described simultaneously in the literature or made available in the market.

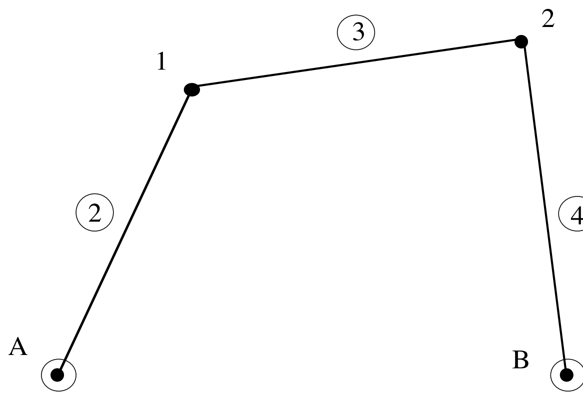

Figure 1.7. Four-bar articulated quadrilateral

1.2 Basic Concepts

1.2.1 Multibody Systems and Joints

We define a *multibody system* as an assembly of two or more rigid bodies (also called elements) imperfectly joined together, having the possibility of relative movement between them. This imperfect joining of the two rigid bodies that makes up a multibody system is called a *kinematic pair* or *joint*, or simply a *joint*. A joint permits certain degrees of freedom of relative motion and prevents or restricts others. A *class I* joint allows one degree of freedom, a *class II* allows two degrees of freedom and so forth. For example, a revolute joint (R) is a *class I* joint that only allows one relative rotation. In planar multibodies, the most used joints are *revolute* (R) and *prismatic* (P), which allow one relative rotation and translation, respectively. In three-dimensional multibodies, *cylindrical* (C), *spherical* (S), *universal* (U), and *helical* (H) joints are also used. Other joints such as *gears* (G) and the *track-wheel rolling contact* (W) are sometimes used. All these will be seen in detail in Chapters 2 and 3.

Usually the elements of a multibody system are linked by means of joints, as shown in the articulated quadrilateral of Figure 1.7. At times, the elements do not have direct contact with one another but rather are interrelated via force transmission elements, such as springs, and shock absorbers or dampers.

Multibody systems are classified as *open-chain* or *closed-chain* systems. If a system is composed of bodies without closed branches (or *loops*), then it is called an open-chain system; otherwise, it is called a closed-chain multibody system. A double pendulum and a tree-type of system are good examples of an open-chain configuration. The four-bar mechanism of Figure 1.7 is an example of a closed-chain system.

1.2.2 Dependent and Independent Coordinates

In order to describe a multibody system, the first important point to consider is that of choosing a mathematical way or model that will describe its position and motion. In other words, select a set of parameters or coordinates that will allow one to unequivocally define the position, velocity, and acceleration of the multibody system at all times. There are several ways to go about solving this problem, and different authors have opted for one way or another depending on their preferences or the peculiarities of their own formulation.

Even though the same multibody system can be described with different types of coordinates, this does not mean that they are all equivalent in the sense that they will allow for formulations that are just as efficient or as easy to implement. In fact, it will be shown in Chapters 2 and 3 that there are differences in computational efficiency and simplicity of implementation when using different sets of coordinates. The different dynamic formulations may also benefit from the characteristics of a particular set of coordinates.

Consequently, the first problem encountered at the time of modeling the motion of a multibody system is that of finding an appropriate system of coordinates. A first choice is that of using a system of *independent coordinates*, whose number coincides with the number of *degrees of freedom* of motion of the multibody system and is thereby minimal. The second choice is to adopt an expanded system of *dependent coordinates* in a number larger than that of the degrees of freedom, which can describe the multibody system much more easily but which are not independent, but interrelated through certain equations known as *constraint equations*. The number of constraints is equal to the difference between the number of dependent coordinates and the number of degrees of freedom. Constraint equations are generally nonlinear, and play a main role in the kinematics and dynamics of multibody systems.

Studies on this subject tend to conclude that independent coordinates are not a suitable solution for a general purpose analysis, because they do not meet one of the most important requirements: that the coordinate system should unequivocally define the position of the multibody system. Independent coordinates directly determine the position of the input bodies or the value of the externally driven coordinates, but not the position of the entire system. Therefore additional non-trivial analysis (seen in Chapter 3) need be performed to this end. For some particular applications, independent coordinates can be very useful to describe with a minimum data set the actual velocities or accelerations and small variations in the position. In addition, they may lead to the highest computational efficiency.

For general cases, the alternative choice to the independent set of coordinates is a system of *dependent coordinates*, which uniquely determine the position of all the bodies. Three major types of coordinates have been proposed to solve this problem: *relative* coordinates, *reference point* or *Cartesian* coordinates, and *natural* or *fully Cartesian* coordinates. The latter are the ones most frequently used in this book. These types of coordinates are described in detail in Chapter

2, both for planar and three-dimensional multibody systems. Although this book deals with these three types of coordinates, it emphasizes the use of the later ones. By means of these coordinates, the position of a three-dimensional object is defined using the Cartesian coordinates of two or more points and the components of one or more unit vectors rigidly attached to the body. Chapters 2 and 3 describe these coordinates in detail, along with other sets of coordinates.

1.2.3 Symbolic vs. Numerical Formulations

Among the computer programs for kinematic and dynamic analysis of multibody systems, there are two groups with very different approaches and capabilities: *symbolic* programs and strictly *numerical* programs.

Symbolic programs do not process numbers from the outset, but variable names and analytical expressions. Their outcome is a list of statements in FORTRAN, C, Pascal, or any other scientific programming language containing the mathematical equations that model the kinematics and/or dynamics of the system in question. If the problem is of large complexity, the formulae can occupy dozens or even hundreds of pages of listings. The advantages of symbolic methods are mainly that they eliminate those operations with variables having zero values, and also allow one to explicitly see the influence of each variable in the equations that control the behavior of the assembly. However, in order for a symbolic code to achieve maximum efficiency it also must be able to compact and simplify the equations by extracting common factors and by compacting trigonometric expressions. Such operations are alleviated through the use of symbolic tools such as MACSYMA, MAPLE, and MATHEMATICA.

Symbolic formulations can be advantageous when the generation of the equations is performed only once and is valid for the entire range of motions that the multibody may undergo. However, one of their major problems stems from the fact that a multibody system may undergo a qualitative change in its kinematic configuration during its motion, thus, demanding a substantial change in the equations of motion. Such situations occur with changes in generalized coordinates, the appearance and disappearance of kinematic constraints, impacts and shocks, backlash, Coulomb friction, etc. Special provisions need to be made in these cases to avoid the complete reformulating of the symbolic equations of motion.

Numerical programs, on the other hand, provide a real general purpose solution to the kinematic and dynamic analysis of all types of multibody systems. These programs formulate the equations of motion numerically without generating analytical equations suited to the specific problem. In many cases, numerical methods are less efficient than the symbolic counterparts. However, their generality and the fact that they are easy to use is a definite advantage. In addition, recent advances in numerical methods have allowed a substantial improvement in the efficiency of numerical approaches and have made them more competitive for many types of applications. These advances include the use of sparse matrix techniques that eliminate operations involving zero terms, and the possibility of using improved dynamic formulations (See Chapter 8).

1.3 Types of Problems

We briefly describe in this section the most important types of kinematic and dynamic problems that occur in real everyday situations.

1.3.1 Kinematic Problems

Kinematic problems are those in which the position or motion of the multibody system are studied, irrespective of the forces and reactions that generate it. Kinematic problems are of a purely geometrical nature and can be solved, irrespective not only of the forces but also of the inertia characteristics of elements such as mass, moments of inertia, and the position of the center of gravity.

We define *input elements* of a multibody system as those whose position or motion is known or specified. The position and motion of the other elements of the system are found in accordance with the position and motion of the input elements. There are as many input elements as there are degrees of freedom for the multibody system. As an example, let us consider the four-bar mechanism of Figure 1.7 in which the crank A-1 (body or element 2) is the input element. Sometimes the kinematic problem is based not on an input element, but on input coordinate or degree of freedom, such as an angle or a distance.

Below is a brief description of the different kinematic problems that occur in practice and which will be discussed in detail in Chapter 3.

Initial Position Problem. The initial position or *assembly* problem consists of finding the position of all the elements of the multibody system once that of the input elements is known. In general, the position problem is difficult to solve, since it leads to a system of nonlinear algebraic equations which has in general several solutions. The more complicated the system is, the larger the number of possible solutions.

Finite Displacement Problem. This problem is a variation of the initial position problem, both from a conceptual point of view as well as from the mathematical methods that can be used to solve it. Given a fixed position on the multibody system and a known finite displacement (not infinitesimal) for the input bodies (or elements), the problem of finite displacements consists of finding the final position of the system's remaining bodies.

In practice, the finite displacement problem ends up being easier to solve than the initial position problem, mainly because one starts from a known position of the system, which can be used as a starting point for the iterative process needed for the solution of the resulting nonlinear equations. The problem of having multiple solutions is not as critical in this case, because usually one is only interested in the solution nearest to the previous position.

Velocity and Acceleration Analysis. Given the position of the multibody system and the velocity of the input elements, velocity analysis consists of determining the velocities of all the other elements and all the points of interest. This problem is much easier to solve than the position problems discussed earlier, mainly because it is linear and has a unique solution.. This means in mathematical terms that it is modeled by a system of linear equations. Consequently, the principle of superposition holds that the velocity of an element is the sum of the velocities produced by each one of the input elements. If all the input velocities are zero, then the velocities of all the elements will also be null.

Given the position and velocity of all the elements in the system, and the acceleration of the input elements, the acceleration analysis consists of finding the acceleration of all the remaining elements and points of interest. Just as the velocity problem is linear, so also is the acceleration problem linear. Moreover, the matrix of the system of linear equations that models this problem is the same as the one in the velocity problem.

Kinematic Simulation. The kinematic simulation provides a view of the entire range of a multibody's motion. The solution of the kinematic simulation encompasses all the previous problems with emphasis on the finite displacement problem. It permits one to detect collisions, study the trajectories of points, sequences of the positions of an element of the multibody system, and the rotation angles of rocker levers and connecting rods, etc.

1.3.2 Dynamic Problems

In general, dynamic problems are much more complicated to solve than kinematics ones. The kinematic problems need be solved before the dynamic problems. Henceforth, it will be assumed that the velocity and acceleration problems can be solved without any difficulty. The outstanding characteristic about dynamic problems is that they involve the forces that act on the multibody system and its inertial characteristics as follows: mass, inertia tensor, and the position of its center of gravity (seen in detail in Chapter 4). We will describe briefly the most important dynamic problems encountered in practice.

Static Equilibrium Position Problem. The static equilibrium position problem (dealt with extensively in Chapter 6) consists of determining the position of the system in which all the gravitational and external forces, elastic forces in the springs, and external reactions are balanced. This problem is not really a dynamic problem, but a static one, that depends on the weight and the position of the center of gravity of the multibody system and not on its inertia properties.

The problem of determining the static equilibrium position takes place very frequently in vehicles with spring suspension systems. It is not always easy (for example, when the loads are not centered) to determine the static equilibrium position at a glance or by means of simple calculations. The general solution to this problem also leads to a system of nonlinear equations which need to be solved iteratively. Even though there might also be several solutions for this case, there are reliable initial estimates normally available that lead to the right solution.

Linearized Dynamics. A problem closely related to the previous one is that of determining the natural vibration modes and frequencies of the small oscillations that take place about the static (or dynamic) equilibrium position. This problem is solved by first linearizing the equations of motion at a particular position, and then performing a step-by-step time history or an eigenvalue analysis. A knowledge of the natural vibration modes and frequencies gives an idea of the system's dynamic stiffness, and it also allows one to design different control systems. This problem is discussed in Chapter 9.

Inverse Dynamic Problem. The inverse dynamic problem aims at determining the motor or driving forces that produce a specific motion, as well as the reactions that appear at each one of the multibody system's joints. It is necessary to know the velocities and accelerations to be able to estimate the inertia forces which, together with the weight, the forces in the springs and dampers and all the other known external forces, will provide the basis to calculate the required actuating forces.

The solution to the inverse dynamics (See Chapter 6) has different applications. In the first place, it determines the forces to which the multibody system is subjected, for both dynamic and kinematic simulation problems. Extremely important is the fact that the inverse dynamics yields the driving forces necessary to control a system so that it follows a desired trajectory.

Forward Dynamic Problem (Dynamic Simulation). The forward dynamic problem yields the motion of a multibody system over a given time interval, as a consequence of the applied forces and given initial conditions. The importance of the direct dynamic problem lies in the fact that it allows one to simulate and predict the system's actual behavior; the motion is always the result of the forces that produce it.

The forward dynamics implies the solution of a system of nonlinear ordinary differential equations (initial value problem). These differential equations are numerically integrated starting from the initial conditions. An important characteristic of this mathematical problem is that it is computationally intensive. Because of this, it is very important to choose the most efficient method for dealing with and solving this problem. The results of the dynamic simulation problem can be displayed numerically, or they can be depicted graphically by means of a plotter of a graphics terminal in the same way as with the kinematic simulation results. Chapter 5 deals with the most common formulations used for dynamic analysis. Chapter 8 presents the most recent ones, including those most suited for real time analysis.

Forward and Inverse Dynamics of Elastic Multibodies. So far we have assumed that all the bodies in a multibody system satisfy the *rigid body condition.* A body is assumed to be rigid if any pair of its material points does not present relative displacements. In practice, bodies suffer some degree of deformation. This tends, however, to be so small that it does affect the system's behavior, and therefore, it can be neglected without commiting an appreciable error.

There are some important cases in which deformation plays an important role in the dynamic analysis. It happens, for instance, in lightweight spatial structures and manipulators, or in high-speed machinery. The complexity and size of the equations of motion considering deformation grow considerably, since all the variables defining the deformation must also be considered. Chapter 11 deals with the forward dynamics of elastic multibodies and Chapter 12 with the inverse dynamics. This case is of particular interest since the driving forces are now non-causal, which means that there is a time delay between actuation and response and the solution goes to negative time and future time.

Percussions and Impacts. Mechanically, a percussion is a force with a large value that occurs in a very short period of time. It is convenient to distinguish between *percussion* and *impact* problems. In the case of percussions, it is assumed that a very large force of known value acts during an infinitesimal amount of time. Bear in mind that the percussion is the value of a mechanical impulse (the integral of the force in relation to time). A typical characteristic of a percussion is that it produces discontinuities (finite jumps) in the distribution of velocities, which are determined from the value of the applied percussion. This problem is of limited practical importance because in practice the percussion value is seldom ever known. The impact problem is more important.

The impact involves the collision of bodies in which at least one of them experiences a sudden change in velocities. The point of contact undergoes a percussion which is generally unknown. In order to be able to calculate the effect of the impact on the system's velocity distribution, it is necessary to introduce an additional equation of experimental nature which measures the nature of the surfaces in contact and the type of impact.

The study of the effects of percussions and impacts in the distribution of velocities of a multibody system can be carried out separately or within a dynamic simulation program. Chapter 10 deals with this problem as well as the design issues outlined in the next section.

1.3.3 Other Problems: Synthesis or Design

We have outlined in the previous sections the most important analysis problems that can occur in the kinematics and dynamics of multibody systems. In all these problems, it is assumed that the geometry and physical properties of the system are known (either because it is an existing multibody system or it has been previously designed). When wishing to design a new system to comply with certain specifications, with only analysis tools available, one must proceed in an iterative manner by means of rough calculations. A preliminary design is carried out and the system is analyzed. Once the results of the analysis have been obtained, the design is then modified if they are not entirely satisfactory, and another analysis is performed. The same procedure is followed until the desired effect is attained. This process may be slow and rather dependent upon the experience of the designer.

Synthesis or design methods help to overcome this difficulty, or at least lessen it. These methods directly lead, without the intervention of an analyst, to a design which complies with the given specifications, or which is the optimal one from a certain design point of view. The design of a multibody system can also be carried out from a more general perspective by taking dynamic factors into account. Two different problems can be considered: pure kinematic design also called *synthesis*, and the more general dynamic sensitivity analysis.

Kinematic Synthesis of Multibody Systems. Kinematic synthesis entails the finding of the best possible dimensions for a given type of multibody system. This is mainly a geometric problem, about which much has been written in the last half of the past century and in the first half of the present one. During this time, many methods were developed, almost all of them graphic and containing a notable amount of ingenuousness and originality. The majority of the methods were focused on the planar four-bar mechanism. However, graphic methods of kinematic synthesis are limited, too specific, and at times difficult to use. In recent years, more general programs based on numerical methods have been developed, and they are applicable to many different types of planar and three-dimensional multibody systems.

Sensitivity Analysis and Optimal Design. The optimal design of a multibody system is started by defining an *objective function* which will optimize the system performance. The solution to the problem will be the configuration that minimizes the objective function. This function is minimized in relation to certain variables which depend on the design of the multibody system and are referred to as *design variables*. It may or may not have *design constraint equations*, that is, equalities or inequalities that should comply with certain specific functions of the design variables. The constraint equations mathematically introduce certain physical design limitations into the problem. An example of a design limitation is that there cannot be any elements with negative mass or length.

The objective functions are defined depending on the application of the multibody system. Since multibody dynamics is a process that takes place over a period of time, it often turns out that the objective function is defined as the time integral of a specific function or as a series of conditions that the multibody must satisfy within certain intervals of time or at specific moments.

There are several optimization methods or ways to minimize the objective function, which are applicable to this problem. Almost all of them are based on the knowledge of the *derivatives* of the objective function with respect to the design variables. The determination of these derivatives is known as *sensitivity* analysis and is the first phase in the optimization process which can also be considered separately. Sensitivity analysis determines the tendencies of the objective function with respect to design variations, and is very useful in a non-automatic interactive design process. Sensitivity analysis is considered in Chapter 10.

1.4 Summary

In this introductory chapter we have tried to outline the different problems that arise in the analysis and design of multibody systems. We have also succinctly commented on the different ways these can be tackled in a very general form, and in this respect it is important to mention that at the present time there is not a formulation or method that can solve all the problems in the best possible manner. Some methods are preferable over others depending on the type of configuration and motion. In the chapters that follow we will deal with all these different problems and try to offer methods for their solution.

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